

Phys. Rev. D 61, 117504 (2000)

## Final-state interaction phase difference in $J/\psi \rightarrow \rho\eta$ and $\omega\eta$ decays

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(February 1, 2008)

### Abstract

It is shown that the study of the  $\omega - \rho^0$  interference pattern in the  $J/\psi \rightarrow (\rho^0 + \omega)\eta \rightarrow \pi^+\pi^-\eta$  decay provides evidence for the large (nearly  $90^\circ$ ) relative phase between the one-photon and the three-gluon decay amplitudes.

13.25.Gv, 11.30.Hv, 13.40.Hq, 14.40.Gx

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In the last few years it has been noted that the single-photon and three-gluon amplitudes in the two-body  $J/\psi \rightarrow 1^-0^-$  and  $J/\psi \rightarrow 0^-0^-$  [1–3] decays appear to have relative phases nearly  $90^\circ$ . This unexpected result is very important to the observability of CP violating decays as well as to the nature of the  $J/\psi \rightarrow 1^-0^-$  and  $J/\psi \rightarrow 0^-0^-$  decays [1–7]. Since the analysis [1–3] involved theoretical assumptions relying on the  $SU_f(3)$  symmetry, the strong  $SU_f(3)$ -symmetry breaking and so on, the measurements of these phases are urgent. Fortunately, it is possible to check the conclusion of Refs. [1,2] at least in one case. We mean the phases between the amplitudes of the one-photon  $J/\psi \rightarrow \rho^0\eta$  and three-gluon  $J/\psi \rightarrow \omega\eta$  decays.

Indeed, the  $\omega - \rho$  interference pattern in  $J/\psi \rightarrow (\rho^0 + \omega)\eta \rightarrow \rho^0\eta \rightarrow \pi^+\pi^-\eta$  is conditioned by the  $\rho^0 - \omega$  mixing and the ratios of the amplitudes of the  $\rho^0$  and  $\omega$  production. As for the  $\rho^0 - \omega$  mixing amplitude, it is reasonably well studied [8–14]. Its modulus and phase are known. The modules of the ratios of the amplitudes of the  $\rho$  and  $\omega$  production can be obtained from the data on the branching ratios of the  $J/\psi$ -decays. So, the interference pattern provides a way of measuring the relative phases of the  $\rho^0$  and  $\omega$  production amplitudes.

The  $\pi^+\pi^-$ -spectrum in the  $\omega, \rho$  energy region is of the form

$$\begin{aligned} \frac{dN}{dm} = N_\rho(m) \frac{2}{\pi} m \Gamma(\rho \rightarrow \pi\pi, m) & \left| \frac{1}{D_\rho(m)} + \frac{\Pi_{\omega\rho^0}(m)}{D_\rho(m)D_\omega(m)} \left[ \frac{N_\omega(m)}{N_\rho(m)} \right]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} + \right. \\ & \left. + \frac{1}{D_\omega(m)} \cdot \frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}} \left[ \frac{N_\omega(m)}{N_\rho(m)} \right]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} \right|^2, \end{aligned} \quad (1)$$

where  $m$  is the invariant mass of the  $\pi^+\pi^-$ -state,  $N_\rho(m)$  and  $N_\omega(m)$  are the squares of the modules of the  $\rho$  and  $\omega$  production amplitudes,  $\delta_\rho$  and  $\delta_\omega$  are their phases,  $\Pi_{\omega\rho^0}(m)$  is the amplitude of the  $\rho - \omega$  transition,  $D_V(m) = m_V^2 - m^2 - im\Gamma_V(m)$ ,  $V = \rho, \omega$ .

In the discussion that follows, Eq. (1) is conveniently rewritten as

$$\begin{aligned} \frac{dN}{dm} = N_\rho(m) \frac{2}{\pi} m \Gamma(\rho \rightarrow \pi\pi, m) & \left| \frac{1}{D_\rho(m)} \left( 1 - \varepsilon(m) \left[ \frac{N_\omega(m)}{N_\rho(m)} \right]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} \right) + \right. \\ & \left. + \frac{1}{D_\omega(m)} (\varepsilon(m) + g_{\omega\pi\pi}/g_{\rho\pi\pi}) \left[ \frac{N_\omega(m)}{N_\rho(m)} \right]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} \right|^2, \end{aligned} \quad (2)$$

where

$$\varepsilon(m) = - \frac{\Pi_{\omega\rho^0}(m)}{m_\omega^2 - m_\rho^2 + im(\Gamma_\rho(m) - \Gamma_\omega(m))}. \quad (3)$$

As known [8–12], the imaginary part of the  $\rho - \omega$  transition amplitude is due to the  $\pi\pi$ ,  $3\pi$ ,  $\gamma\pi$  and  $\gamma\eta$  intermediate states

$$\begin{aligned} \text{Im}(\Pi_{\rho^0\omega}(m)) = m & \left( \frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}} \Gamma(\rho \rightarrow \pi\pi, m) + \frac{g_{\rho\rho\pi}}{g_{\omega\rho\pi}} \Gamma(\omega \rightarrow \rho\pi \rightarrow 3\pi, m) + \right. \\ & \left. + \frac{g_{\rho\gamma\pi}}{g_{\omega\gamma\pi}} \Gamma(\omega \rightarrow \gamma\pi, m_\omega) + \frac{g_{\rho\gamma\eta}}{g_{\omega\gamma\eta}} \Gamma(\omega \rightarrow \gamma\eta, m) \right). \end{aligned} \quad (4)$$

The quite conservative estimate of the contribution of the  $\pi\pi$  and  $3\pi$  intermediate states gives

$$\begin{aligned} m_\omega \frac{g_{\omega\pi\pi}}{g_{\rho\pi\pi}} \Gamma(\rho \rightarrow \pi\pi, m_\omega) &= \pm m_\omega \cdot 10^{-2} \cdot \Gamma(\rho \rightarrow \pi\pi, m_\omega) = \pm 1.17 \cdot 10^{-3} \text{ GeV}^{-2}, \\ m_\omega \frac{g_{\rho\rho\pi}}{g_{\omega\rho\pi}} \Gamma(\omega \rightarrow \rho\pi \rightarrow 3\pi, m_\omega) &= \pm m_\omega \cdot 10^{-2} \cdot \Gamma(\omega \rightarrow 3\pi, m_\omega) = \pm 5.84 \cdot 10^{-5} \text{ GeV}^{-2}. \end{aligned} \quad (5)$$

The constituent quark and vector meson dominance models both give the same result

$$\begin{aligned} m_\omega \frac{g_{\rho\gamma\pi}}{g_{\omega\gamma\pi}} \Gamma(\omega \rightarrow \gamma\pi, m_\omega) &= m_\omega \cdot \frac{1}{3} \cdot \Gamma(\omega \rightarrow \gamma\pi, m_\omega) = 1.86 \cdot 10^{-4} \text{ GeV}^{-2}, \\ m_\omega \frac{g_{\rho\gamma\eta}}{g_{\omega\gamma\eta}} \Gamma(\omega \rightarrow \gamma\eta, m_\omega) &= m_\omega \cdot 3 \cdot \Gamma(\omega \rightarrow \gamma\eta, m_\omega) = 1.28 \cdot 10^{-5} \text{ GeV}^{-2}. \end{aligned} \quad (6)$$

Notice that the predictions of the constituent quark and vector meson dominance models on the  $\omega \rightarrow \gamma\pi(\eta)$  and  $\omega \rightarrow \gamma\pi(\eta)$  decays agree adequately with the experiment.

As is seen from Eqs. (3) and (4), the contribution of the  $\pi\pi$  intermediate state in  $Im(\Pi_{\omega\rho^0}(m))$  and the  $g_{\omega\pi\pi}$  direct coupling constant cancel considerably in the  $g_{\omega\pi\pi}^{eff}$  effective coupling constant:

$$\begin{aligned} g_{\omega\pi\pi}^{eff}(m) &= \varepsilon(m) \cdot g_{\rho\pi\pi} + g_{\omega\pi\pi} = -\frac{\Pi'_{\omega\rho^0}(m) \cdot g_{\rho\pi\pi} + i\Gamma(\rho \rightarrow \pi\pi, m) \cdot g_{\omega\pi\pi}}{m_\omega^2 - m_\rho^2 + im(\Gamma_\rho(m) - \Gamma_\omega(m))} + g_{\omega\pi\pi} = \\ &= -\frac{\Pi'_{\omega\rho^0}(m) + (m_\rho^2 - m_\omega^2 + im\Gamma_\omega(m)) \cdot (g_{\omega\pi\pi}/g_{\rho\pi\pi})}{m_\omega^2 - m_\rho^2 + im(\Gamma_\rho(m) - \Gamma_\omega(m))} \cdot g_{\rho\pi\pi} = \\ &= -\frac{\Pi'_{\omega\rho^0}(m) \mp 1.87 \cdot 10^{-4} \text{ GeV}^{-2} \pm i6.6 \cdot 10^{-5} \text{ GeV}^{-2}}{m_\omega^2 - m_\rho^2 + im(\Gamma_\rho(m) - \Gamma_\omega(m))} \cdot g_{\rho\pi\pi}, \end{aligned} \quad (7)$$

where  $\Pi'_{\omega\rho^0}(m)$  is the amplitude of the  $\rho^0 - \omega$  transition without the contribution of the  $\pi\pi$  intermediate state in the imaginary part, the numerical values are calculated at  $m = m_\omega$ .

The branching ratio of the  $\omega \rightarrow \pi\pi$  decays

$$B(\omega \rightarrow \pi\pi) = \frac{\Gamma(\rho \rightarrow \pi\pi, m_\omega)}{\Gamma_\omega(m_\omega)} \cdot |\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}|^2 \quad (8)$$

It follows from Eqs. (6) and (7) that imaginary part of the numerator in Eq. (7) is dominated by the  $\gamma\pi$  intermediate state to within 35%. This imaginary part gives  $B(\omega \rightarrow \pi\pi) \simeq 5 \cdot 10^{-5}$  instead of the experimental value [14]

$$B(\omega \rightarrow \pi^+\pi^-) = 0.0221 \pm 0.003. \quad (9)$$

So, one can get the modulus of the real part of the numerator in Eq. (7) which is clearly dominated by  $Re(\Pi_{\omega\rho^0}(m))$ . Besides, the interference pattern of the  $\rho^0$  and  $\omega$  mesons in the  $e^+e^- \rightarrow \pi^+\pi^-$  reaction and in the  $\pi^+\pi^-$  photoproduction on nuclei shows [8–12] that the real part of the numerator in Eq. (7) is positive. So, from Eqs. (3), (7), (8) and (9) one obtains

$$Re(\Pi_{\rho^0\omega}(m_\omega)) = (3.80 \pm 0.27) \cdot 10^{-3} \text{ GeV}^2 \quad (10)$$

and

$$\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi} = (3.41 \pm 0.24) \cdot 10^{-2} \exp\{i(102 \pm 1)^\circ\} . \quad (11)$$

The data [15,16] were fitted with the function

$$N(m) = L(m) + \left| (N_\rho)^{\frac{1}{2}} F_\rho^{BW}(m) + (N_\omega)^{\frac{1}{2}} F_\omega^{BW}(m) \exp\{i\phi\} \right|^2 , \quad (12)$$

where  $F_\rho^{BW}(m)$  and  $F_\omega^{BW}(m)$  are the appropriate Breit-Wigner terms [15] and  $L(m)$  is a polynomial background term.

The results are

$$\begin{aligned} \phi &= (46 \pm 15)^\circ , \quad N_\omega(m_\omega)/N_\rho = 8.86 \pm 1.83 \quad [15] , \\ \phi &= -0.08 \pm 0.17 = (-4.58 \pm 9.74)^\circ , \quad N_\omega(m_\omega)/N_\rho = 7.37 \pm 1.72 \quad [16] . \end{aligned} \quad (13)$$

From Eqs. (2), (8), and (12) follows

$$N_\rho = N_\rho(m_\rho) \left| 1 - \varepsilon(m_\rho) [N_\omega(m_\rho)/N_\rho(m_\rho)]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} \right|^2 , \quad (14)$$

$$N_\omega = B(\omega \rightarrow \pi\pi) N_\omega(m_\omega) , \quad (15)$$

$$\begin{aligned} \phi &= \delta_\omega - \delta_\rho + \arg[\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}] - \\ &- \arg\left\{ 1 - \varepsilon(m_\rho) [N_\omega(m_\rho)/N_\rho(m_\rho)]^{\frac{1}{2}} \exp\{i(\delta_\omega - \delta_\rho)\} \right\} \simeq \\ &\simeq \delta_\omega - \delta_\rho + \arg[\varepsilon(m_\omega) + g_{\omega\pi\pi}/g_{\rho\pi\pi}] - \arg\left\{ 1 - |\varepsilon(m_\omega)| [N_\omega(m_\omega)/N_\rho]^{\frac{1}{2}} \exp\{i\phi\} \right\} . \end{aligned} \quad (16)$$

From Eqs. (11), (13) and (16) we obtain

$$\delta_\rho - \delta_\omega = \delta_\gamma = (60 \pm 15)^\circ \quad [15] , \quad (17)$$

$$\delta_\rho - \delta_\omega = \delta_\gamma = (106 \pm 10)^\circ \quad [16] . \quad (18)$$

A large (nearly  $90^\circ$ )  $\delta_\gamma$  was obtained in Ref. [1,2]. So, both the MARK III Collaboration [15] and the DM2 Collaboration [16], see Eqs. (17) and (18), provide support for this view.

The DM2 Collaboration used statistics only half as high as the MARK III Collaboration, but, in contrast to the MARK III Collaboration, which fitted  $N_\omega$  as a free parameter, the DM2 Collaboration calculated it from the branching ratio of  $J/\psi \rightarrow \omega\eta$  using Eq. (15).

In summary we would like to emphasize that it would be beneficial to study this fundamental problem once again with BES in Beijing.

We thank G.F. Xu very much for discussions. The present work was supported in part by the grant INTAS-RFBR IR-97-232.

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